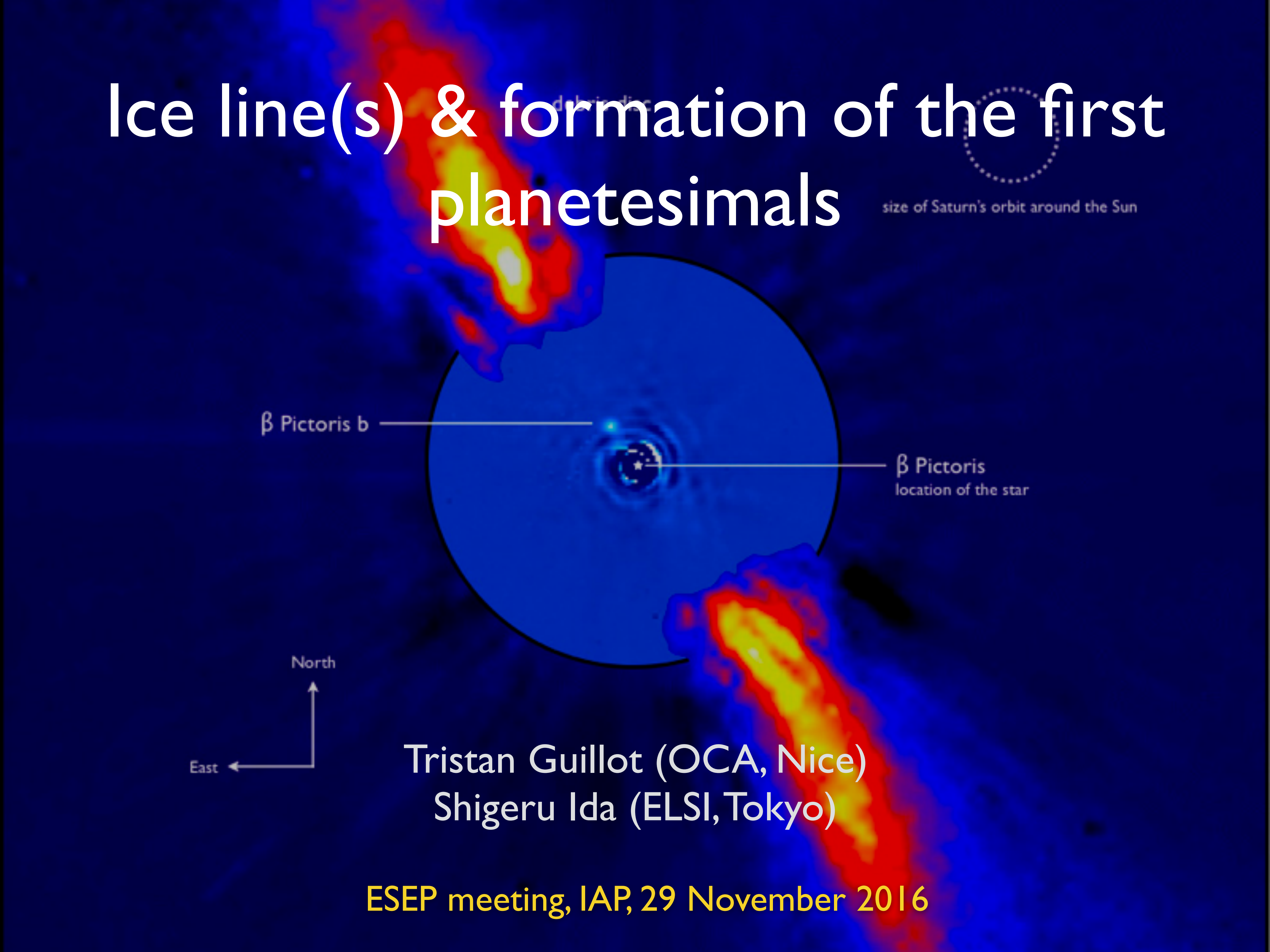
Ice line(s) & formation of the first planetesimals



Tristan Guillot (OCA, Nice)
Shigeru Ida (ELSI, Tokyo)

ESEP meeting, IAP, 29 November 2016

Solar System Formation: Astrophysical background

50 Ma

Solar System Formation: Astrophysical background

0 Ma

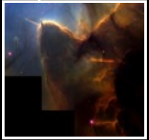
50 Ma

Solar System Formation: Astrophysical background

0 Ma

The protosun and the protosolar disk form

50 Ma



Solar System Formation: Astrophysical background

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Solar System Formation: Astrophysical background

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gas disk phase

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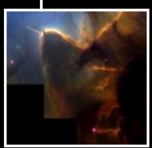

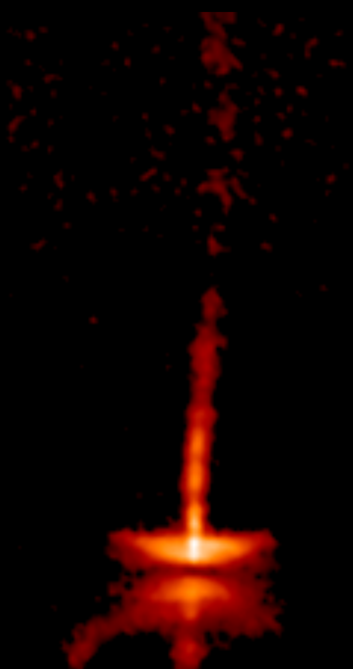

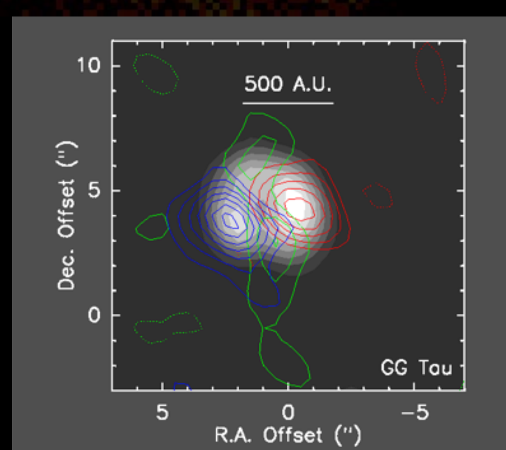

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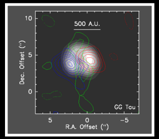

Solar System Formation: Astrophysical background

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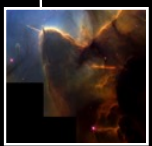

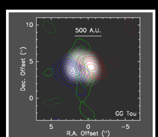

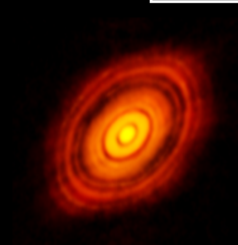

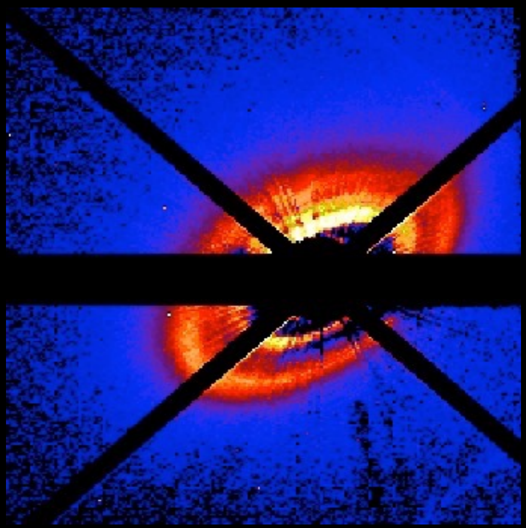

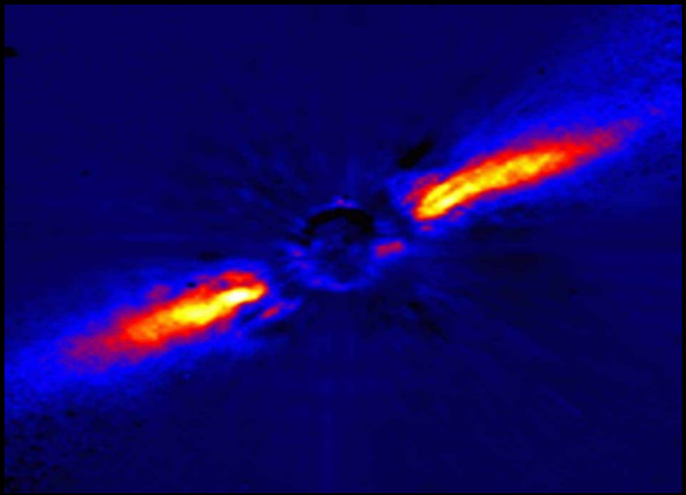

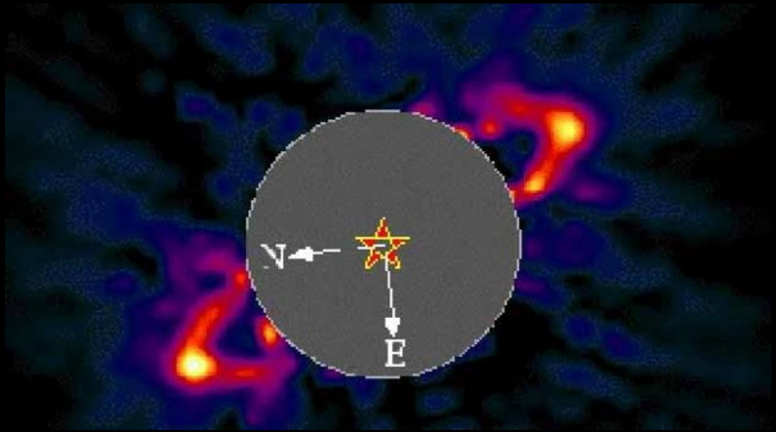

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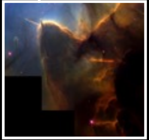

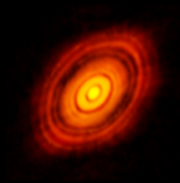

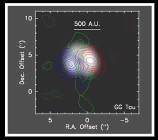

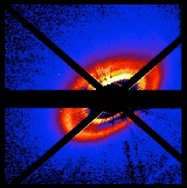

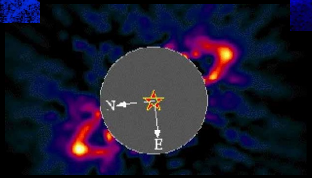

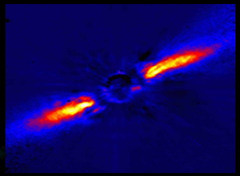

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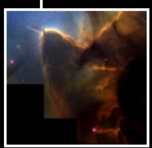

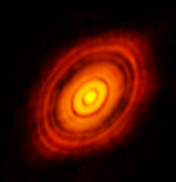

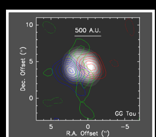

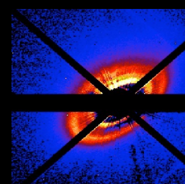

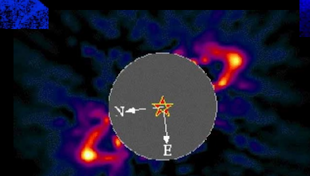

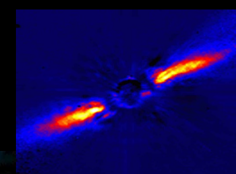

Giant planets form

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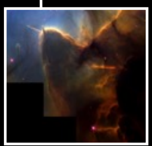


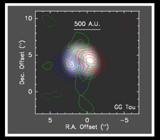
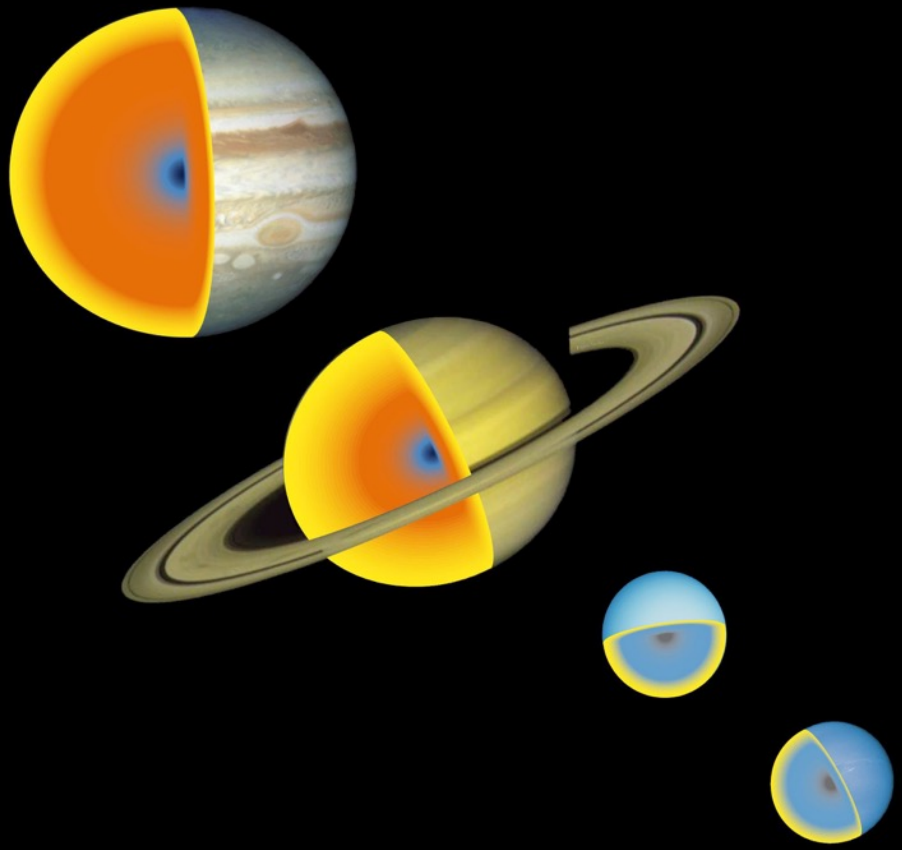


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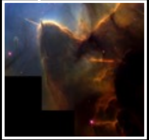

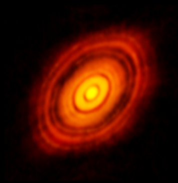

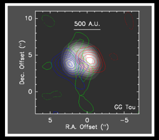

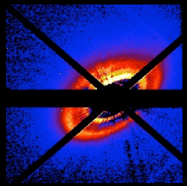

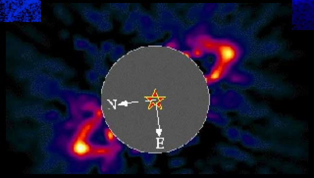

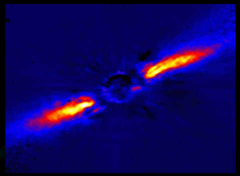

Giant planets form

Terrestrial planets form

Standard picture

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- A circumstellar disk form from the collapse of a molecular cloud core and spreads viscously (e.g., Shakura & Sunyaev 1973, Lynden-Bell & Pringle 1974, Shu 1977)
    - The collapse of the cloud takes $\sim 10^5$ yrs, disk spreading takes 10^6 to 10^8 yrs.

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- A circumstellar disk form from the collapse of a molecular cloud core and spreads viscously (e.g., Shakura & Sunyaev 1973, Lynden-Bell & Pringle 1974, Shu 1977)
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- Planetesimals (1-10km) form rapidly (e.g., Weidenschilling 1980)
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- Oligarchic growth (Kokubo & Ida 1998, Thommes et al. 2003)
  - Slower growth of oligarchs by accretion of smaller embryos.
  - This phase ends when the mass in small planetesimals has become too small to damp the eccentricities of large embryos. This occurs for masses between moon mass at 1 au and up to 10 M_{Earth} at 10 au, on timescales of ~10^5 yrs to several 10^6 yrs.

Beyond the standard picture

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- Planetesimals do not form easily
  - Solids cannot form a small-enough mid-plane for gravitational instabilities in the dust to form planetesimals directly (Dubrulle et al. 1995)
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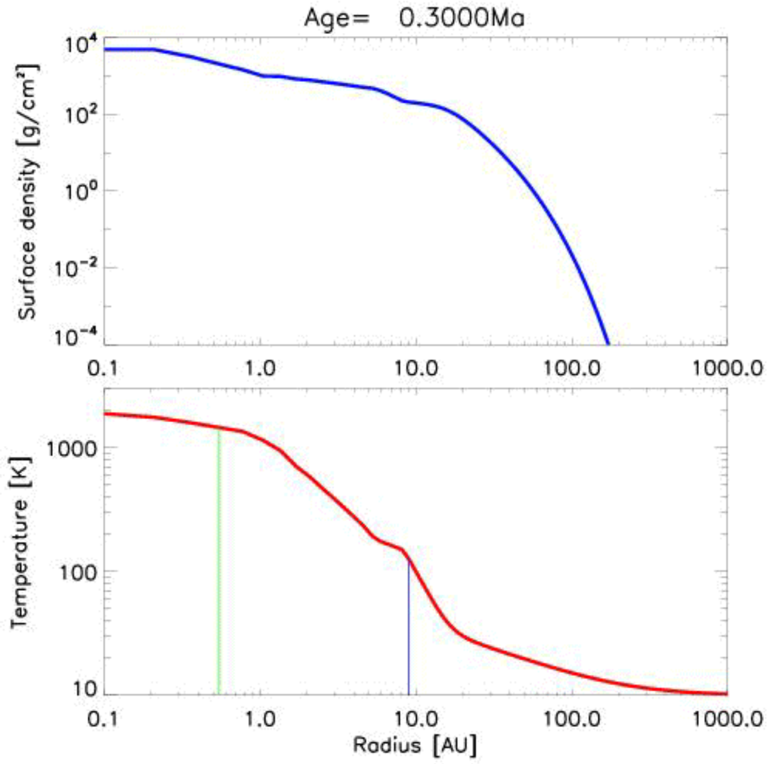

Hueso & Guillot (2005)

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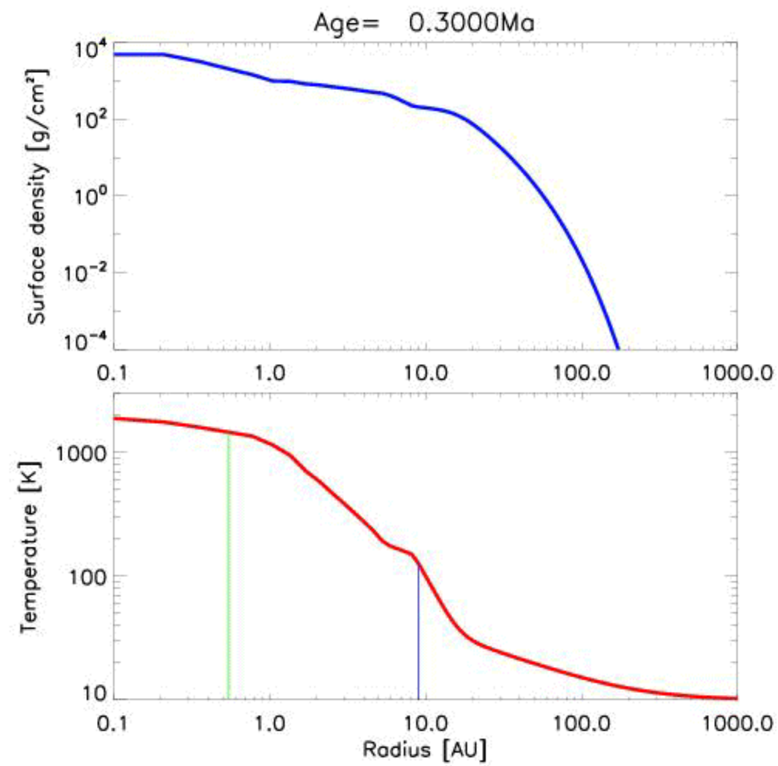

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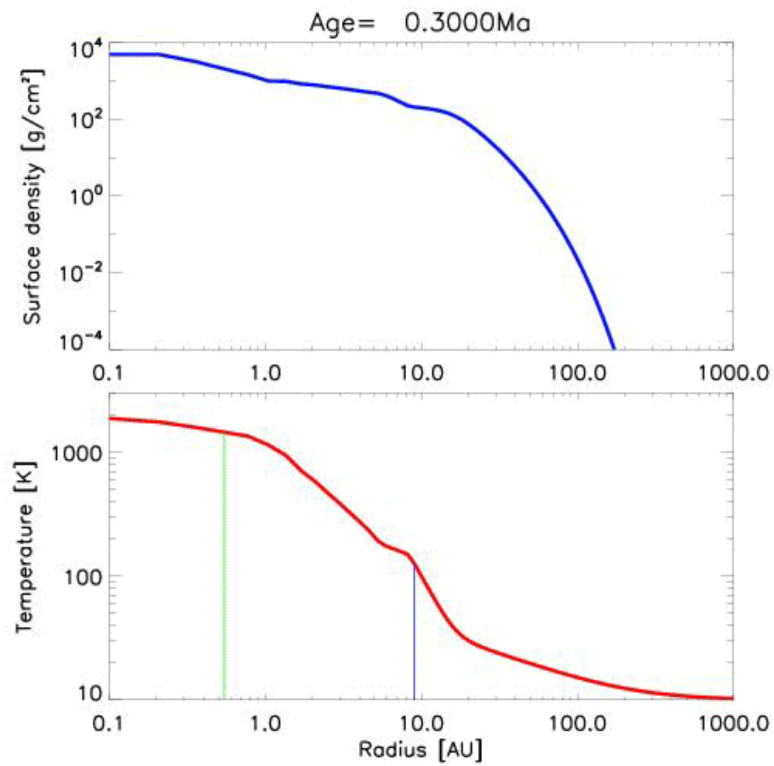

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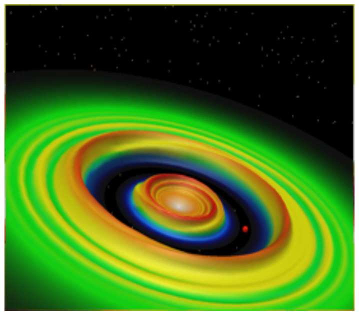

Dobbs-Dixon et al. (2006)

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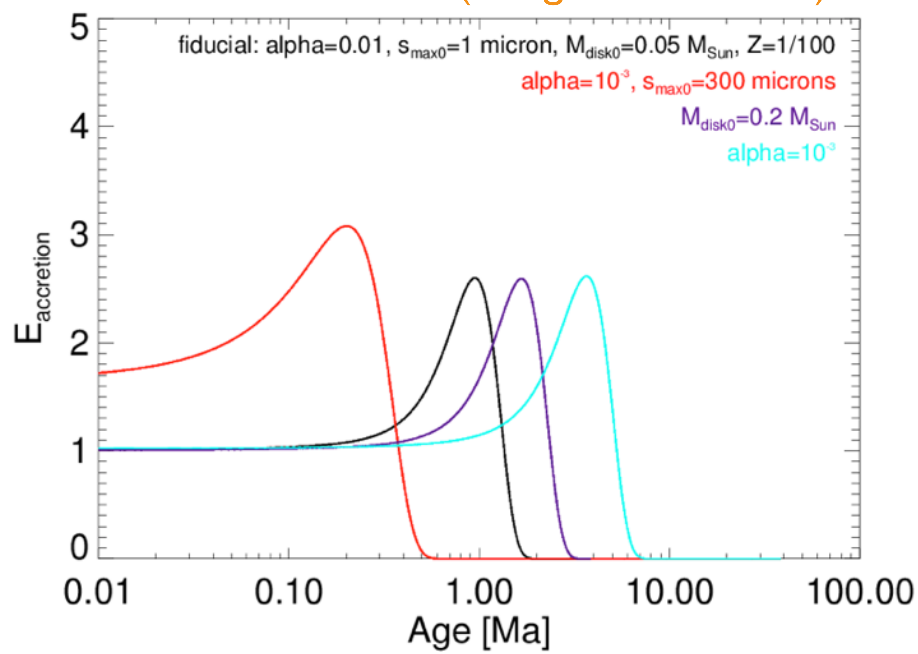
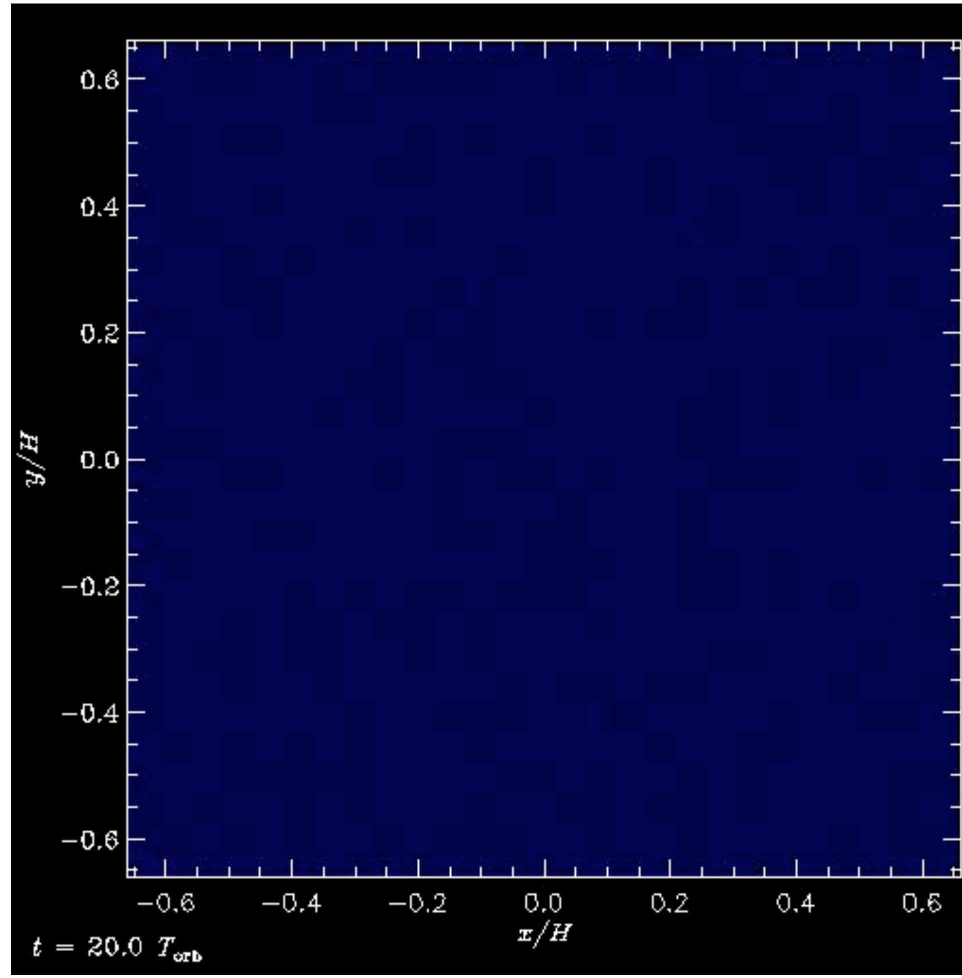
Evolution of the composition of accreted material (using Garaud 2007)



Forming planetesimals: streaming instabilities



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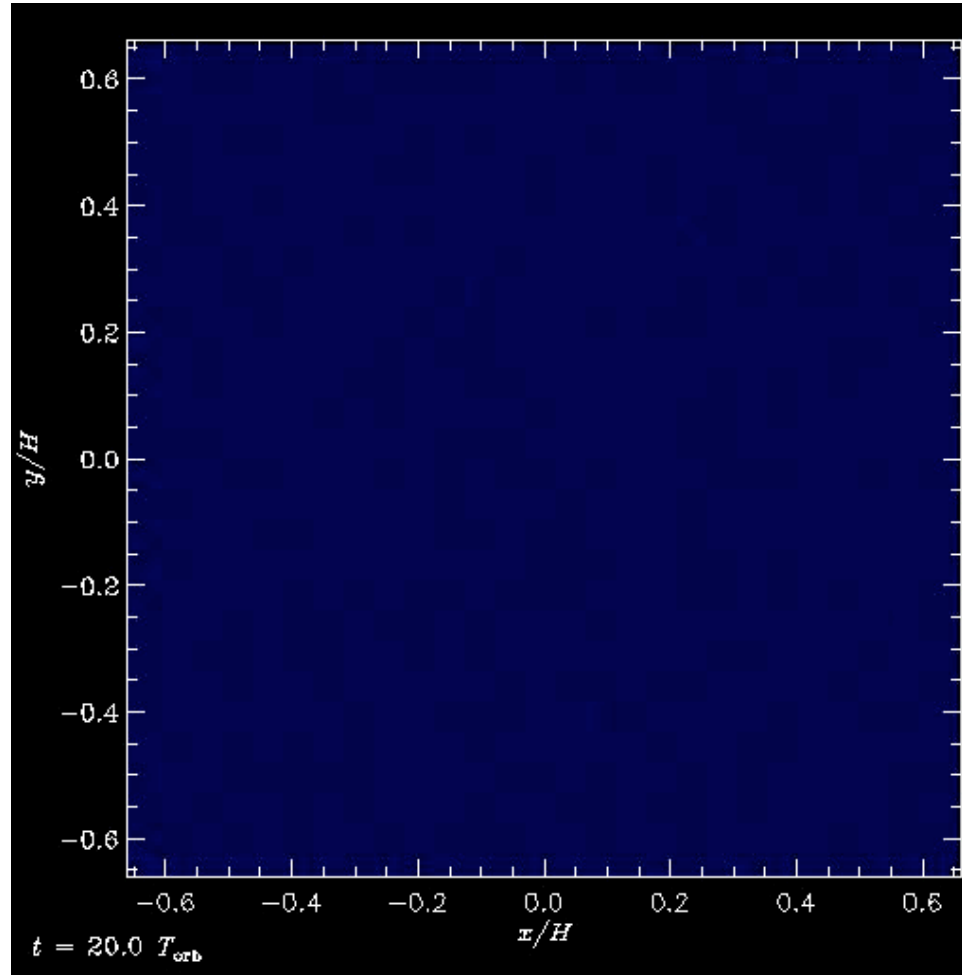
- Planetesimals may be formed rapidly (~orbital periods) by streaming instabilities (Youdin & Goodman 2005, Johansen et al. 2007)



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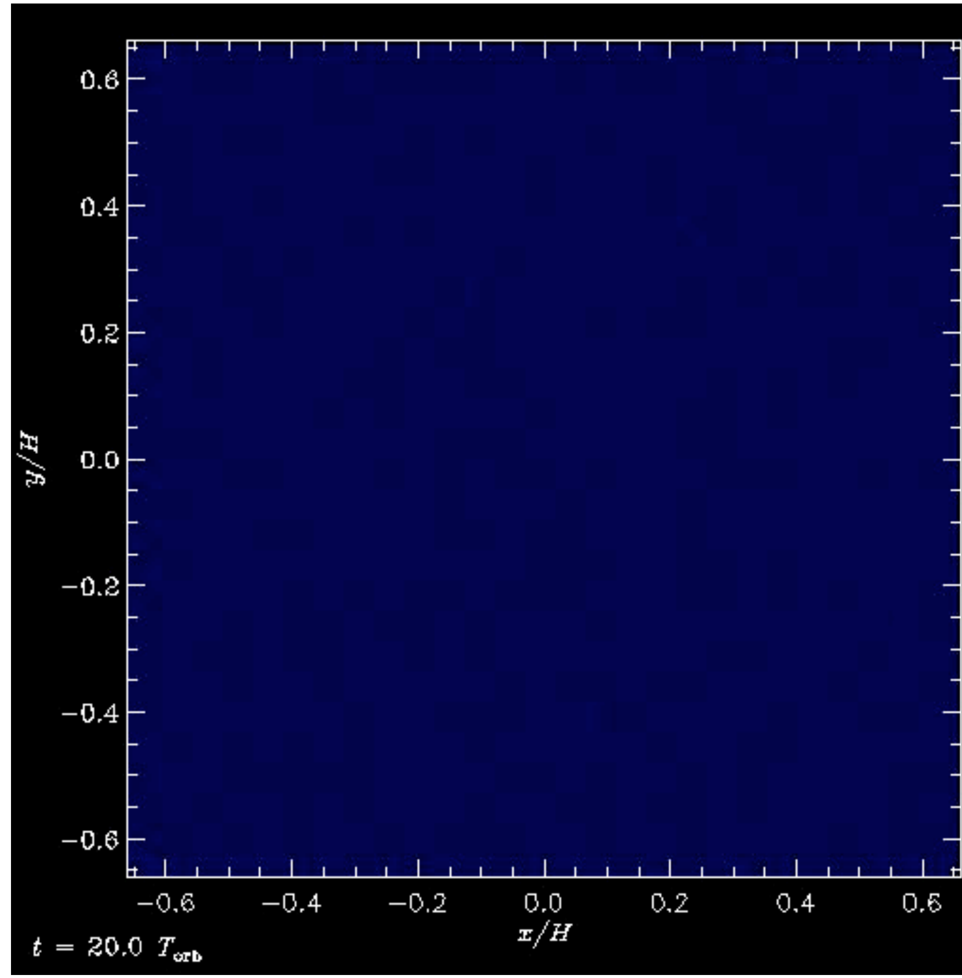

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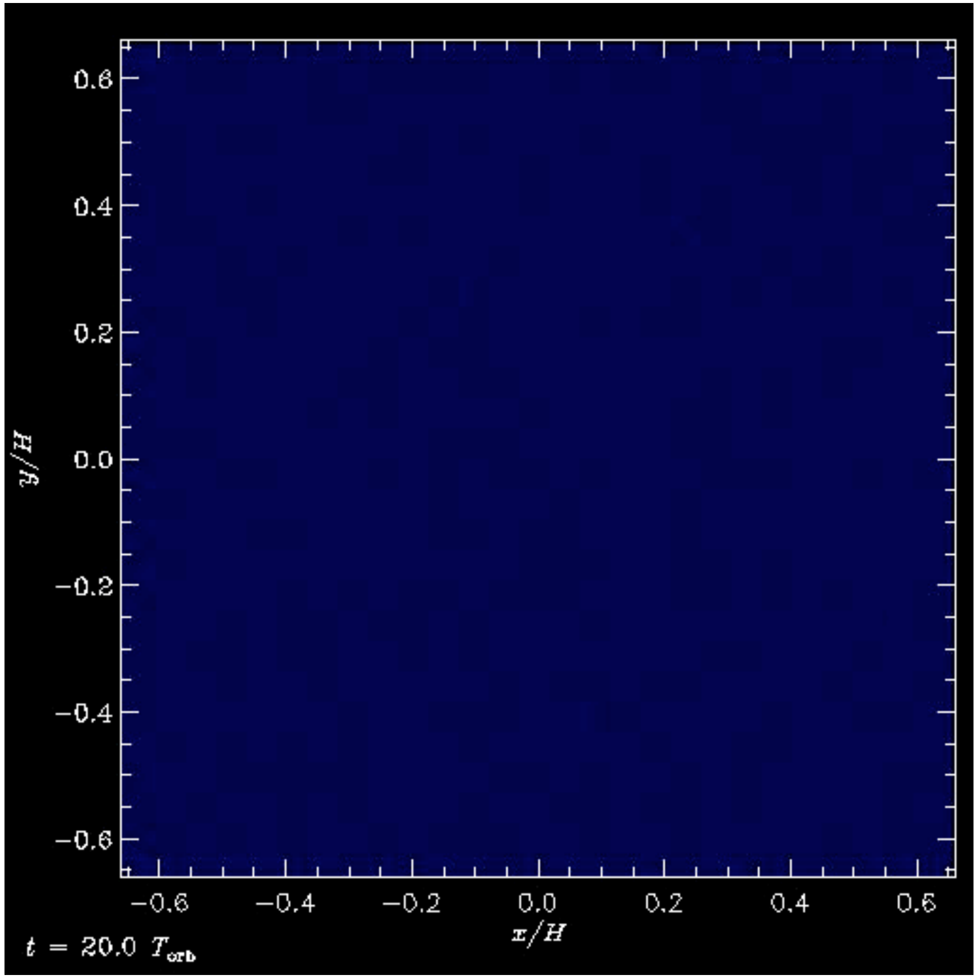

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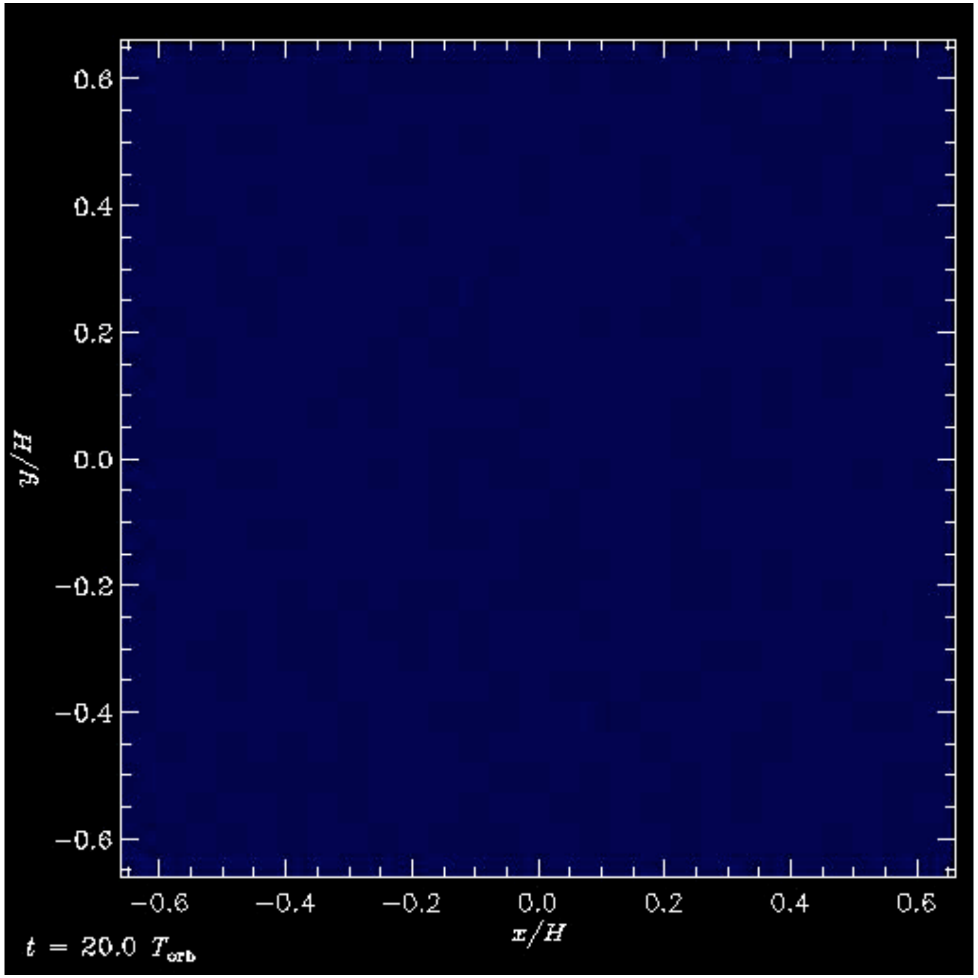
- These instabilities may lead to the direct formation of Ceres-mass objects (Johansen et al. 2011)
- However, this requires high enrichments, difficult to reach for pebbles (see Carrera et al. 2015; Krijt et al. 2016)



Johansen et al. (2011)

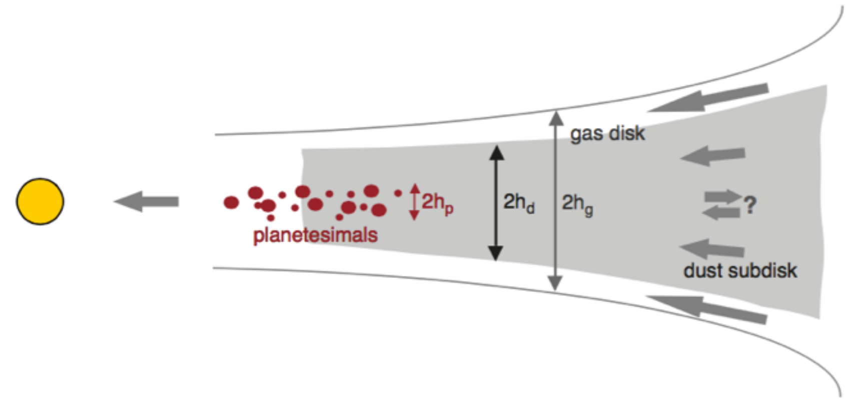
Ida & Guillot, A&A 596, L3 (2016)

Formation of dust-rich planetesimals
from sublimated pebbles inside of the snow line

Forming the first planetesimals from pebbles

$$\Sigma_{\rm p} = \dot{M}_{\rm peb}/2\pi r v_r,$$

$$\Sigma_{\rm g} \simeq \dot{M}_*/3\pi\nu \simeq \dot{M}_*/3\pi\alpha h_{\rm g}^2\Omega_{\rm K},$$



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The inward radial drift speed of solids was calculated in the limit of a static disk by Nakagawa et al. (1986) and in the limit of a low solid-to-gas ratio by Guillot et al. (2014). Combining the two yields

$$v_r = -\Lambda^2 \frac{2\tau_{\rm s}}{1+\Lambda^2\tau_{\rm s}^2}\eta v_{\rm K} + \Lambda\frac{1}{1+\Lambda^2\tau_{\rm s}^2}u_\nu, \quad (2)$$

where the back-reaction of the gas to the motion of solids has been included through $\Lambda \equiv \rho_{\rm g}/(\rho_{\rm g}+\rho_{\rm p})$, and $\rho_{\rm g}$ and $\rho_{\rm p}$ are the mid-plane densities of gas and solids, respectively. We included the Λ-dependence of u_ν as well for later purposes. In Eq. (2), the size of the solids is defined through their Stokes number $\tau_{\rm s}$, which is the ratio of their stopping time due to gas drag ($t_{\rm stop}$) to the Kepler frequency as

$$\tau_{\rm s} = t_{\rm stop}\Omega_{\rm K}, \quad (3)$$

u_ν is the radial velocity of the accreting disk gas, which in the inner regions of a vertically uniform disk may be approximated by

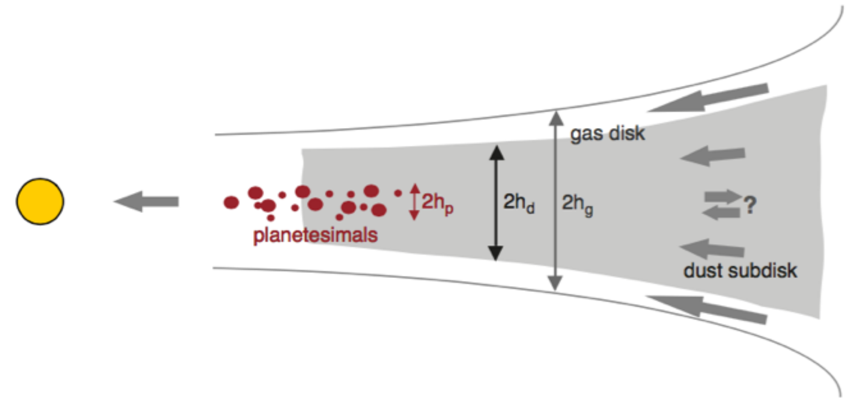
$$u_\nu \sim -3\nu/2r \sim -3\alpha h_{\rm g}^2\Omega_{\rm K}/2r \sim -(3/2)\alpha(h_{\rm g}/r)^2 v_{\rm K}, \quad (4)$$

and η ($\ll$1) is the deviation fraction of the gas orbital angular velocity (Ω) relative to the Keplerian angular velocity ($\Omega_{\rm K}$) that is due to the radial pressure gradient in the disk,

$$\eta = \frac{\Omega_{\rm K}-\Omega}{\Omega_{\rm K}} \simeq \frac{1}{2}\left(\frac{h_{\rm g}}{r}\right)^2\left(-\frac{{\rm d}\ln P}{{\rm d}\ln r}\right). \quad (5)$$

From Eqs. (2), (4), and (5),

$$v_r \simeq -\frac{1}{1+\Lambda^2\tau_{\rm s}^2}\left[\Lambda^2\tau_{\rm s}\left(-\frac{{\rm d}\ln P}{{\rm d}\ln r}\right)+\Lambda\frac{3\alpha}{2}\right]\left(\frac{h_{\rm g}}{r}\right)^2 v_{\rm K}. \quad (6)$$



Using Eq. (1), we then obtain the solid-to-gas ratio as

$$Z = \frac{\Sigma_{\rm p}}{\Sigma_g} \simeq \frac{3\alpha}{2}\left(\frac{h_{\rm g}}{r}\right)^2\frac{v_{\rm K}}{v_r}\frac{\dot{M}_{\rm peb}}{\dot{M}_*}$$
$$\simeq \left(1+\Lambda^2\tau_{\rm s}^2\right)\left[\frac{2\tau_{\rm s}}{3\alpha}\Lambda^2\left(-\frac{{\rm d}\ln P}{{\rm d}\ln r}\right)+\Lambda\right]^{-1}\frac{\dot{M}_{\rm peb}}{\dot{M}_*}. \quad (7)$$

Now, the parameter Λ may be estimated in the limit of a vertically isothermal disk as

$$\Lambda^{-1} = 1 + \frac{\Sigma_{\rm p}/h_{\rm p}}{\Sigma_{\rm g}/h_{\rm g}} = 1 + Z\frac{h_{\rm g}}{h_{\rm p}}. \quad (8)$$

This thus leads to the following second-order equation in Z:

$$(\xi\beta^2-\beta)Z^2 - (A+1-2\xi\beta)Z + (1+\tau_s^2)\xi = 0, \quad (9)$$

and ξ is the ratio of the solid mass flux to the gas mass flux:

$$\xi \equiv \dot{M}_{\rm peb}/\dot{M}_*. \quad (12)$$

Forming the first planetesimals from pebbles

$$\Sigma_{\rm p} = \dot{M}_{\rm peb}/2\pi r v_r,$$
$$\Sigma_{\rm g} \simeq \dot{M}_*/3\pi\nu \simeq \dot{M}_*/3\pi\alpha h_{\rm g}^2\Omega_{\rm K},$$

The inward radial drift speed of solids was calculated in the limit of a static disk by Nakagawa et al. (1986) and in the limit of a low solid-to-gas ratio by Guillot et al. (2014). Combining the two yields

$$v_r = -\Lambda^2\frac{2\tau_{\rm s}}{1+\Lambda^2\tau_{\rm s}^2}\eta v_{\rm K} + \Lambda\frac{1}{1+\Lambda^2\tau_{\rm s}^2}u_\nu, \quad (2)$$

where the back-reaction of the gas to the motion of solids has been included through $\Lambda \equiv \rho_{\rm g}/(\rho_{\rm g}+\rho_{\rm p})$, and $\rho_{\rm g}$ and $\rho_{\rm p}$ are the mid-plane densities of gas and solids, respectively. We included the Λ-dependence of u_ν as well for later purposes. In Eq. (2), the size of the solids is defined through their Stokes number $\tau_{\rm s}$, which is the ratio of their stopping time due to gas drag ($t_{\rm stop}$) to the Kepler frequency as

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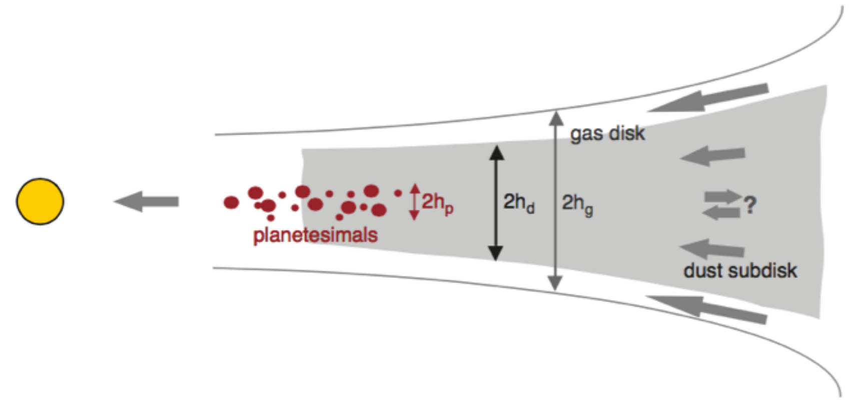
$$u_\nu \sim -3\nu/2r \sim -3\alpha h_{\rm g}^2\Omega_{\rm K}/2r \sim -(3/2)\alpha(h_{\rm g}/r)^2 v_{\rm K}, \quad (4)$$

and η ($\ll$1) is the deviation fraction of the gas orbital angular velocity (Ω) relative to the Keplerian angular velocity ($\Omega_{\rm K}$) that is due to the radial pressure gradient in the disk,

$$\eta = \frac{\Omega_{\rm K}-\Omega}{\Omega_{\rm K}} \simeq \frac{1}{2}\left(\frac{h_{\rm g}}{r}\right)^2\left(-\frac{{\rm d}\ln P}{{\rm d}\ln r}\right). \quad (5)$$

From Eqs. (2), (4), and (5),

$$v_r \simeq -\frac{1}{1+\Lambda^2\tau_{\rm s}^2}\left[\Lambda^2\tau_{\rm s}\left(-\frac{{\rm d}\ln P}{{\rm d}\ln r}\right)+\Lambda\frac{3\alpha}{2}\right]\left(\frac{h_{\rm g}}{r}\right)^2 v_{\rm K}. \quad (6)$$



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Forming the first planetesimals from pebbles

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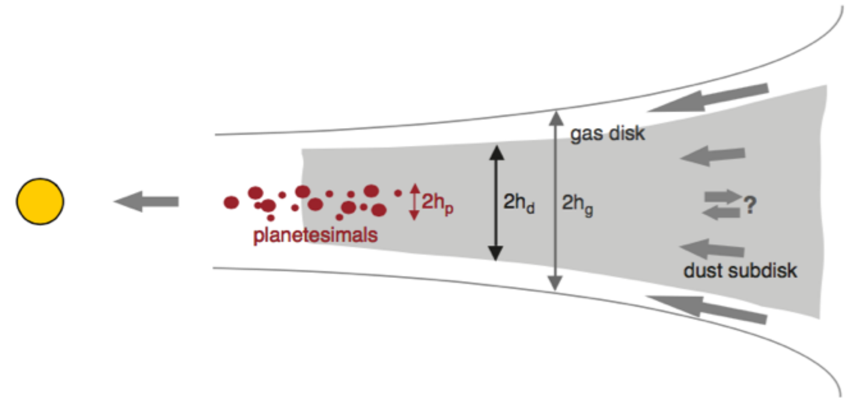

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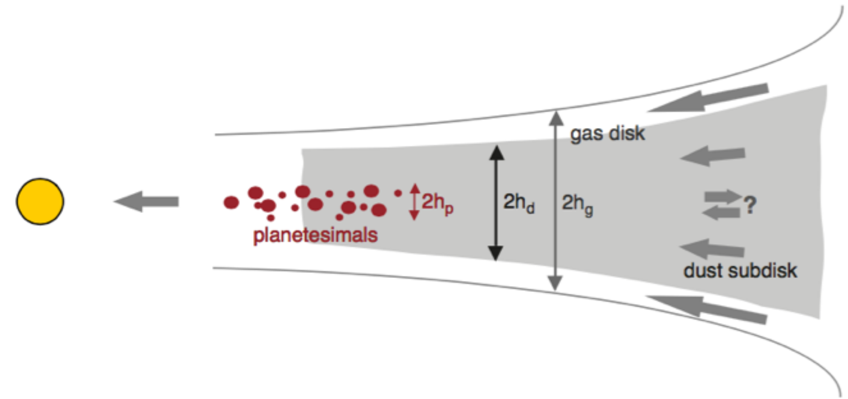

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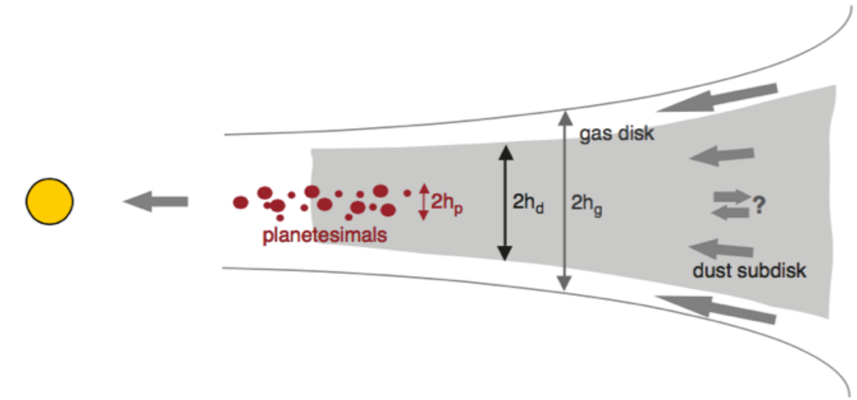

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Z vs. Tau_s

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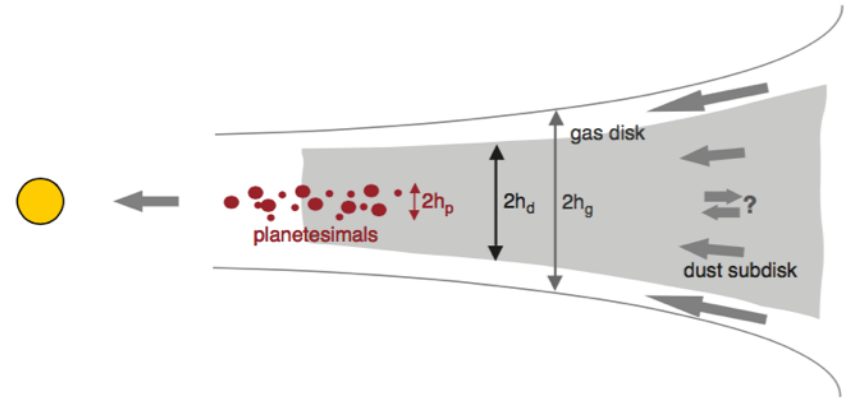

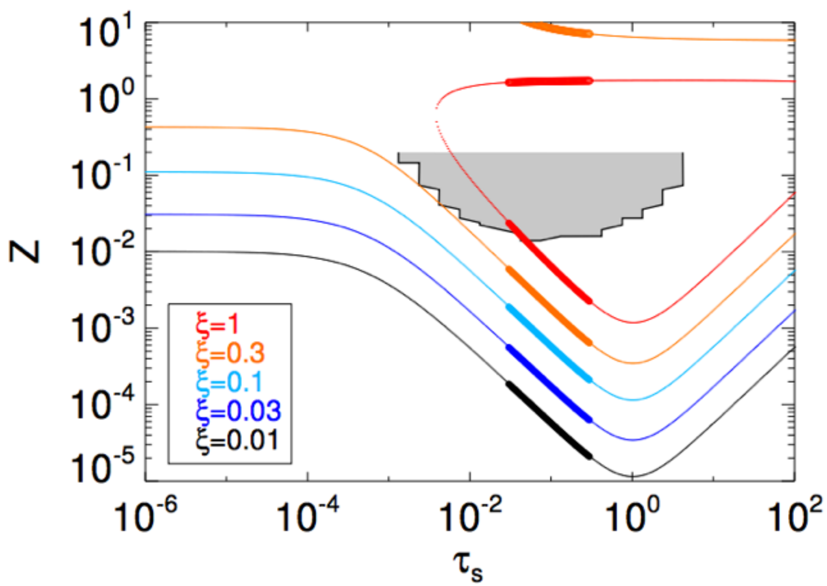

Fig. 1. Steady-state solutions for the solid-to-gas mixing ratio Z as a function of the Stokes number of solid particles τ_s for different values of the solid-to-dust-mass flux ratio ξ (as labeled), assuming a value of the turbulent viscosity $\alpha = 10^{-3}$. The values of τ_s corresponding to expected pebble sizes are highlighted with larger symbols. The two solutions provided by Eq. (13) are indicated by filled and open symbols, respectively. The gray area highlights the region in which planetesimals should form by a streaming instability (Carrera et al. 2015).

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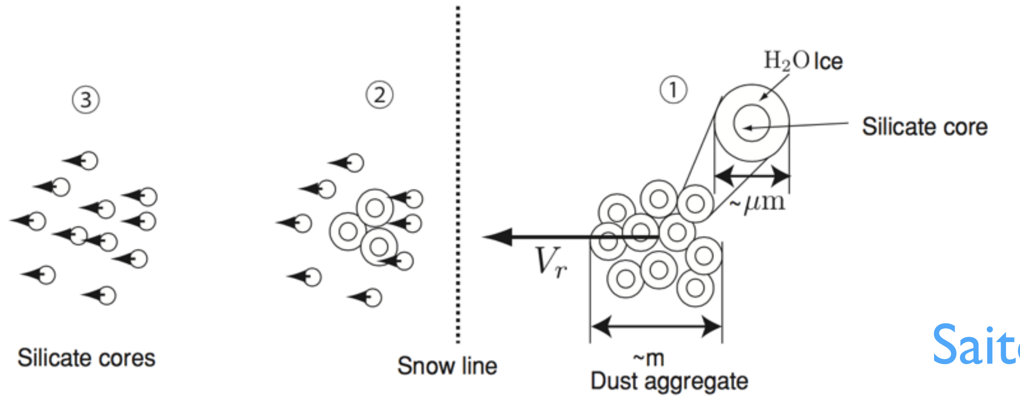
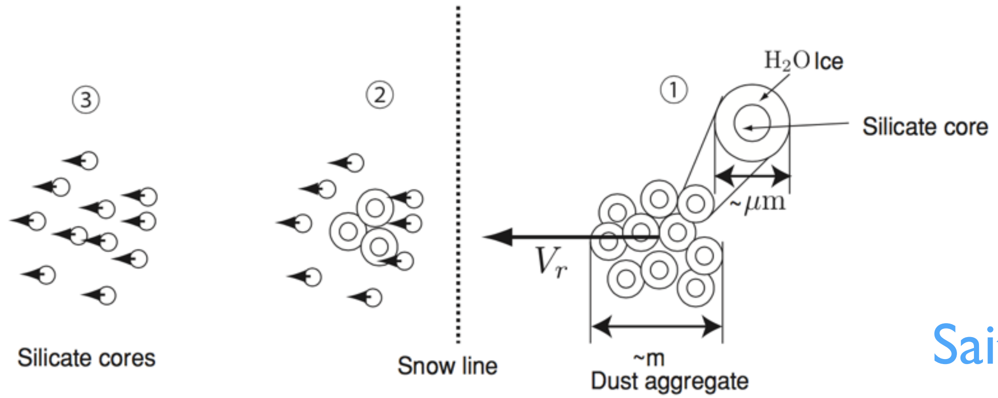
Inside of the snow line



Saito & Sirono (2011)

Inside of the snow line



Saito & Sirono (2011)

A slow vertical mixing:

Second, we expect dust grains to retain a memory of the vertical scale height of the pebbles. Their vertical mixing timescale can be estimated to be

$$t_{\rm mix} \sim (h_{\rm g}/\ell)^2 \Omega_{\rm K}^{-1} \sim 160\alpha_3^{-1} T_{\rm K} \sim 160\alpha_3^{-1}(r/1\,{\rm au})^{3/2}\ {\rm yr}, \quad (16)$$

where $\ell \sim \sqrt{\alpha} h_{\rm g}$ is the estimated vertical mean free path and $T_{\rm K}$ is Kepler period. Comparing a sublimation timescale with a migration timescale for pebbles, we can derive the radial width for completion of the sublimation as $\Delta r \sim 10^{-2}(R/10\,{\rm cm})^{1/2} r$ (see also Ciesla & Cuzzi 2006). With Eq. (1), the timescale for the pebble flux to establish $Z \gtrsim 1$ in the sublimation region is estimated as $t_Z \sim 2\pi r \Delta r \Sigma_{\rm g}/\dot{M}_{\rm peb} \sim (1/3\pi)(r/h_{\rm g})^2(\Delta r/r)\alpha^{-1}\xi_{\rm peb}^{-1} T_{\rm K} \sim 10^3(R/10\,{\rm cm})^{1/2}\alpha_3^{-1}\xi_{\rm peb}^{-1} T_{\rm K}$. Although t_Z for $R = 10$ cm is 10–100 times longer than $t_{\rm mix}$, the effective R for sublimation would be much smaller and t_Z would be much shorter for more realistic fluffy pebbles (e.g., Kataoka et al. 2013). We can thus assume that the dust seeds released by the sublimating pebbles have the same vertical thickness as the pebbles themselves. This is done in Eq. (13) by replacing β by the value set by the pebble subdisk $\beta \rightarrow \beta_0 \sim (1 + \tau_{\rm s,peb}/\alpha)^{1/2}$.

Inside of the snow line

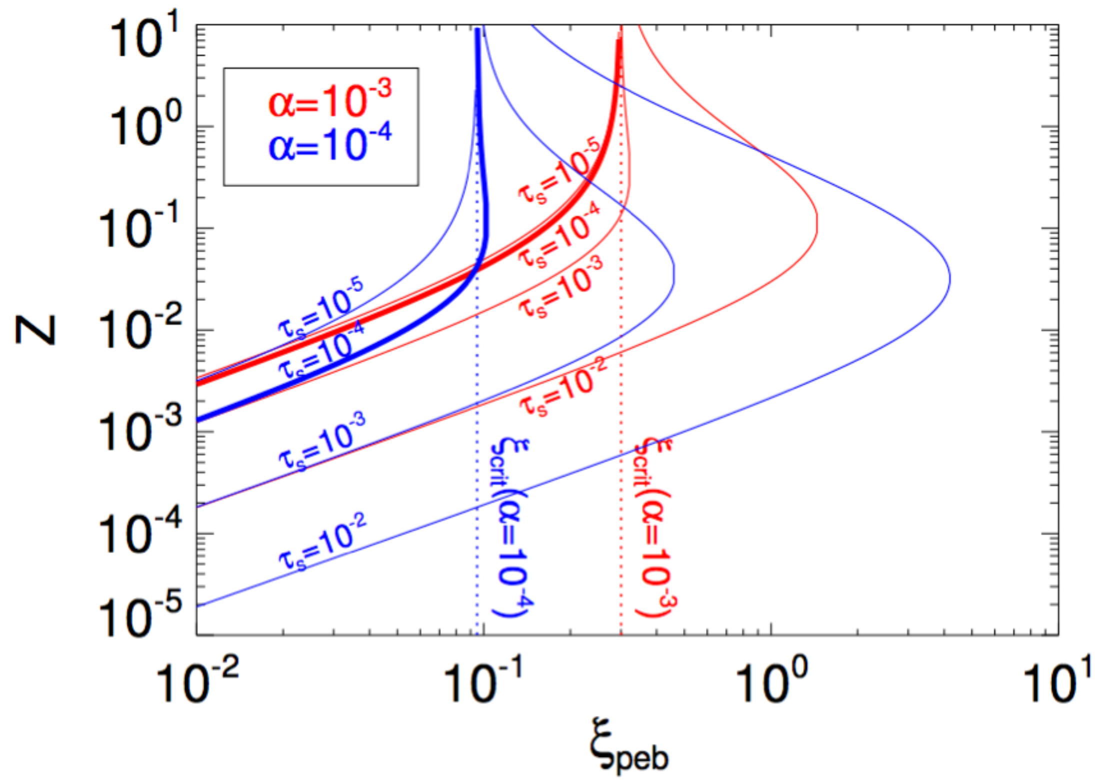


Fig. 2. Steady-state solutions for the solid-to-gas mixing ratio Z as a function of the solid-to-gas-mass flux ratio ξ for different values of the Stokes number of solid particles τ_s (from 10^{-5} to 10^{-2}, as labeled), assuming two values of the turbulent viscosity $\alpha = 10^{-4}$ (in blue) and $\alpha = 10^{-3}$ (in red). Equation (13) is numerically solved. In contrast to Fig. 1, we now consider that initially icy pebbles with $\tau_{s,peb} \sim 0.1$ and containing a mass fraction $\zeta_0 = 1/3$ in dust sublimate inside of the snow line. The thicker lines corresponds to the preferred value for the dust particles, $\tau_s = 10^{-4}$.

Estimating ξ

$$\dot{M}_{\rm peb} \simeq 2\pi r_{\rm peb} \times Z_0 \Sigma_g(r_{\rm peb}) \times \frac{{\rm d}r_{\rm peb}}{{\rm d}t},$$

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The pebble formation front is defined either by the growth timescale of pebbles

$$t_{\rm grow} \sim 10 \times \frac{4}{\sqrt{3\pi}} \frac{1}{Z_0 \Omega} \sim 210\, Z_{02}^{-1} M_{*0}^{-1/2} \left(\frac{r_{\rm peb}}{1\ {\rm au}}\right)^{3/2}\ {\rm yr},$$

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Or by the location at which the disk is gravitationally unstable in the gas

$$Q(r_{\rm GI}/h_{\rm g})^3 \simeq 3\alpha(M_*/\dot{M}_*)\Omega,$$

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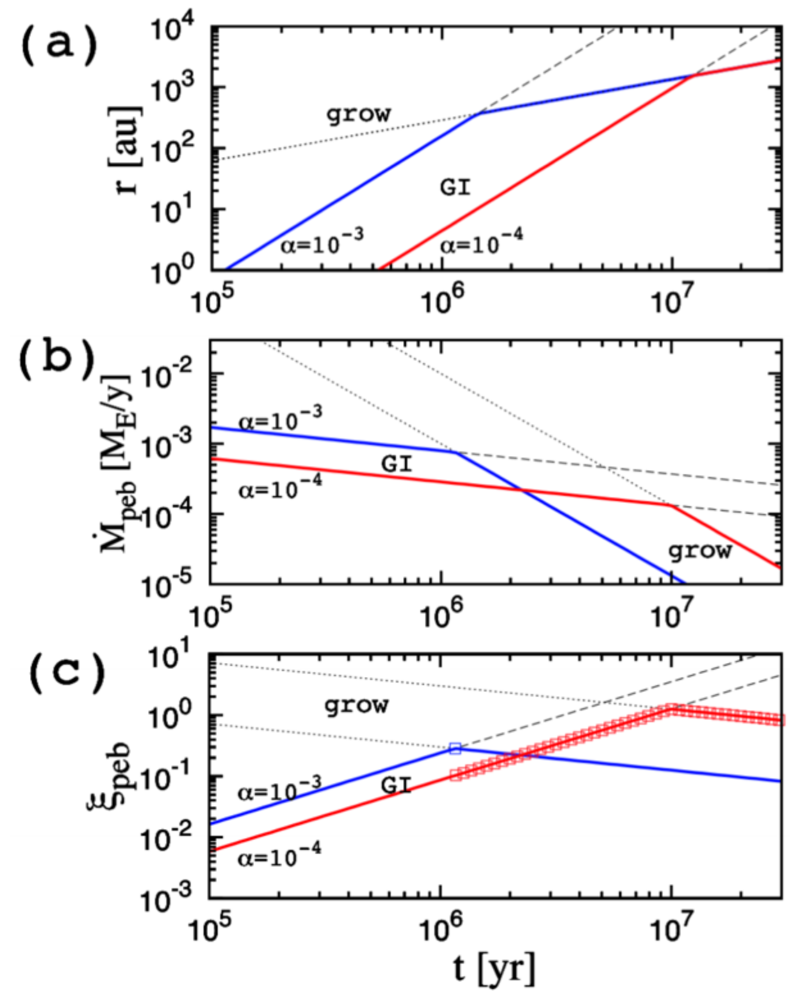

Fig. 3. Time evolution of **a)** radius of the pebble formation front, **b)** pebble accretion rate ($\dot{M}_{\rm peb}$), and **c)** $\xi_{\rm peb} = \dot{M}_{\rm peb}/\dot{M}_*$ for two values of α, 10^{-4} (red) and 10^{-3} (blue). The lines labeled "grow" (dotted) and "GI" (dashed) represent the pebble growth and disk GI limits, respectively. The thick solid lines express the actual values obtained by the minima of the two limits. Here we assumed $\tau_{\rm s,peb} = 0.1$ and $Z_0 = 0.01$. In panel **c)**, the small squares represent the points with $\xi_{\rm peb} > \xi_{\rm crit}$, see Eq. (18).

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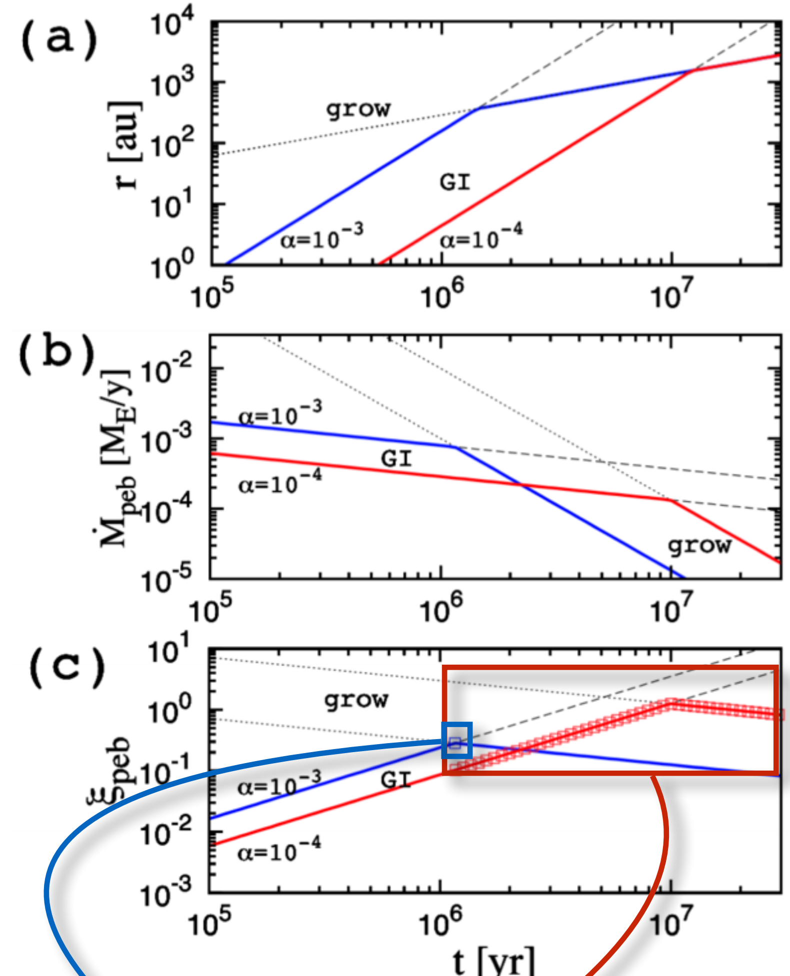

Fig. 3 Time evolution of a) radius of the pebble formation front, b) peb-

Formation of dust-rich planetesimals directly by gravitational instability

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Conclusions

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  - dust-to-gas ratio is low due to the fast migration of pebbles
    - Streaming instability may be possible at high turbulence levels (e.g., $\alpha>0.01$)
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- Including the back reaction of the solids on the gas yields an instability for small particles and high pebble fluxes
  - the instability can occur when the pebble to gas mass flux ratio exceeds $0.1(\alpha/10^{-4})^{1/2}$
    - This instability is favored at low turbulence levels

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    - Streaming instability may be possible at high turbulence levels (e.g., $\alpha>0.01$)
  - one possibility would be to include pressure bumps
- Including the back reaction of the solids on the gas yields an instability for small particles and high pebble fluxes
  - the instability can occur when the pebble to gas mass flux ratio exceeds $0.1(\alpha/10^{-4})^{1/2}$
    - This instability is favored at low turbulence levels
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  - Ice sublimation transforms pebbles into small (dust-rich) grains
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Conclusions

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- These planetesimals are **dust-rich**
  - **How do solar abundances relate to the ice-to-rock ratio in the solar system?**
  - **What about that Ice-to-rock ratio in exoplanets?**